Eliassen–Palm Fluxes and the Momentum Equation in Non-Eddy-Resolving Ocean Circulation Models

PETER R. GENT

National Center for Atmospheric Research, Boulder, Colorado

JAMES C. MCWILLIAMS

National Center for Atmospheric Research, Boulder, Colorado and Institute of Geophysics and Planetary Physics, University of California, Los Angeles, Los Angeles, California

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ABSTRACT

The concepts of residual-mean circulation, transformed Eulerian-mean equations, and Eliassen–Palm fluxes are generalized when the averaging is a low-pass operator in time and space rather than a zonal average. Thus, the eddy motions being considered are ocean eddies on short time and small space scales rather than either purely transient eddies or steady, zonally averaged, standing eddies as commonly considered for the atmosphere.

The generalized Eliassen–Palm fluxes are then parameterized as downgradient momentum diffusion plus the appropriate Coriolis term. This gives a momentum equation for use in non-eddy-resolving ocean circulation models. The resulting potential vorticity equation is then analyzed and the quasigeostrophic limit taken. When the adiabatic tracer parameterization of Gent and McWilliams is also used, this equation is close to showing that quasigeostrophic potential vorticity is advected by the geostrophic velocity and diffused by a Laplacian operator.

A discussion of the Antarctic Circumpolar Current and the meridional-plane circulation, the Deacon cell, in the Southern Hemisphere ocean follows. In an eddy-resolving model with nearly adiabatic interior dynamics, the Deacon cell essentially does not appear when the zonal averaging of the meridional velocity is taken along a constant density surface. This result has a counterpart in non-eddy-resolving ocean model simulations in that the Deacon cell is partially cancelled by the parameterized eddy-induced mass transport.

1. Introduction

The expression Eliassen–Palm fluxes has been used since Andrews and McIntyre (1976) based their theory on the pioneering work of Eliassen and Palm (1961). This theory describes the interaction between standing waves and the zonal mean flow especially in the stratosphere where transience, forcing, and dissipation are relatively weak. The theory also generalizes the pioneering work of Charney and Drazin (1961) by showing when the nonacceleration theorem is valid. Then eddy effects on the mean circulation can be completely described by the transformed Eulerian-mean equations defined in terms of the residual-mean circulation. A lucid account of these concepts is in the book by Andrews et al. (1987).

Gent and McWilliams (1990) and Gent et al. (1995) propose an adiabatic parameterization (GM) for the effects of mesoscale eddies on the transport of tracers in a non-eddy-resolving ocean circulation model. The parameterization consists of an additional advection of tracers by the eddy-induced transport velocity and the diffusion of tracers along surfaces of constant potential density. A form for the eddy-induced velocity is proposed in Gent and McWilliams (1990). The results of this parameterization in non-eddy-resolving ocean models have been shown to be generally beneficial for tracer distributions and their fluxes; see Danabasoglu et al. (1994), Böning et al. (1995), Danabasoglu and McWilliams (1995), Robitaille and Weaver (1995), England (1995), Hirst and McDougall (1996a,b, manuscript submitted to *J. Phys. Oceanogr.*), and Visbeck et al. (1996, manuscript submitted to *J. Phys. Oceanogr.*). The conventional momentum equation was used in these studies; should a differently parameterized equation be used in non-eddy-resolving models?

In this note, we bring these two lines of work together. In section 2 we define a residual-mean circulation and transformed Eulerian-mean equations that have zonal and temporal variations. The GM parameterization can then be interpreted as a statement about the density equation in this set as well as a prescription of the residual-mean circulation. In section 3 we parameterize the resulting Eliassen–Palm momentum fluxes in terms of the mean fields to obtain the form of

Corresponding author address: Dr. Peter R. Gent, National Center for Atmospheric Research, P.O. Box 3000, Boulder, CO 80307-3000. E-mail: gent@ncar.ucar.edu

the momentum equation for non-eddy-resolving models. Then the consequences of this parameterization for the kinetic energy and potential vorticity budgets are explored. Section 4 is a discussion of the Antarctic Circumpolar Current and the Deacon cell in the Southern Hemisphere ocean, as well as the relation between these phenomena in eddy-resolving and non-eddy-resolving ocean circulation models. Our conclusions are presented in section 5.

There has been much recent independent work in these same areas. Lee and Leach (1996) propose Eliassen–Palm fluxes appropriate for time mean, rather than zonal mean, flows in isopycnal coordinates. They then show that the divergence of these fluxes is close to the eddy fluxes of a linearized potential vorticity, which generalizes the well-known quasigeostrophic result. McDougall and McIntosh (1996a,b, manuscripts submitted to *J. Phys. Oceanogr.*) propose a temporal residual-mean velocity suitable for the time-averaged, rather than zonally averaged, tracer equation in a non-eddy-resolving ocean circulation model. This is a generalization of the GM parameterization. Tandon and Garrett (1996) analyze a two-dimensional jet and show that the mean meridional circulation should disappear. They show that it cannot disappear without a parameterization such as GM. How the present work relates to these independent contributions will be explained in the later sections.

2. Generalized Eliassen–Palm fluxes

Consider incompressible, Boussinesq, hydrostatic, and adiabatic flow in Cartesian coordinates. The governing equations are

$$\frac{D}{Dt}\mathbf{u} + f\mathbf{k}\times\mathbf{u} + \nabla p = \mathcal{D}, \tag{1}$$

$$\frac{D}{Dt}\rho = 0, \tag{2}$$

$$p_z + g\rho/\rho_0 = 0, \tag{3}$$

$$\nabla\cdot\mathbf{u} + w_z = 0, \tag{4}$$

where D/Dt is the substantial derivative, which advects with the horizontal and vertical velocities $(\mathbf{u}, w)$; ∇ is the horizontal gradient operator, ρ is the density that is assumed here to be the same as the potential density, and p is the pressure divided by a reference density ρ_0; $\mathcal{D}$ is a small-scale smoothing operator without which these equations cannot be integrated because enstrophy accumulates on the grid scale, no matter how fine. The generalization of the issues we discuss below to the more complex thermodynamics of seawater is addressed in Gent and McWilliams (1990) and Gent et al. (1995).

The variables are now decomposed into low-pass components, denoted by an overbar, and eddy components by a low-pass filtering operator in time and space. We will not define this operator explicitly, and will use the notation $\widehat{ab}$ to denote the filtered residual quantity $\overline{ab} - \bar{a}\bar{b}$. As opposed to an average over a coordinate, a filtered quantity does not exclude quadratic interactions between filtered and unfiltered components. Thus, its parameterization is considerably more subtle and difficult. In this paper the eddy terms play an essentially symbolic role until they are given specific parameterization forms in the next section. Some discussion below does pertain to averages over a coordinate, but the appropriate overbar definition is then explicitly given. The filtered equations (1) and (2) have eddy terms: for example, the filtered zonal component of (1) is

$$\frac{D}{Dt}\bar{u} + \nabla\cdot(\widehat{\mathbf{u}u}) + (\widehat{wu})_z - f\bar{v} + \bar{p}_x = 0, \tag{5}$$

where $\mathcal{D}$ has, for convenience, been absorbed into the eddy terms. Now define velocities

$$\mathbf{U} = \bar{\mathbf{u}} - (\widehat{\mathbf{u}\rho}/\bar{\rho}_z)_z, \tag{6}$$

$$W = \bar{w} + \nabla\cdot(\widehat{\mathbf{u}\rho}/\bar{\rho}_z), \tag{7}$$

and the modified substantial derivative, D^*/Dt, which advects with $(\mathbf{U}, W)$. Then the filtered equations (1) and (2) can be written in the form

$$\frac{D^*}{Dt}\bar{u} - fV + \bar{p}_x = \nabla^{3D}\cdot\mathbf{E}, \tag{8}$$

$$\frac{D^*}{Dt}\bar{v} + fU + \bar{p}_y = \nabla^{3D}\cdot\mathbf{F}, \tag{9}$$

$$\frac{D^*}{Dt}\bar{\rho} = -G_z, \tag{10}$$

where ∇^{3D} is the 3D gradient operator, and

$$\mathbf{E} = [\bar{u}_z\widehat{u\rho}/\bar{\rho}_z - \widehat{uu}, \bar{u}_z\widehat{v\rho}/\bar{\rho}_z - \widehat{vu}, -\bar{u}_x\widehat{u\rho}/\bar{\rho}_z + (f - \bar{u}_y)\widehat{v\rho}/\bar{\rho}_z - \widehat{wu}], \tag{11}$$

$$\mathbf{F} = [\bar{v}_z\widehat{u\rho}/\bar{\rho}_z - \widehat{uv}, \bar{v}_z\widehat{v\rho}/\bar{\rho}_z - \widehat{vv}, -(f + \bar{v}_x)\widehat{u\rho}/\bar{\rho}_z - \bar{v}_y\widehat{v\rho}/\bar{\rho}_z - \widehat{wv}], \tag{12}$$

$$G = \bar{\rho}_x\widehat{u\rho}/\bar{\rho}_z + \bar{\rho}_y\widehat{v\rho}/\bar{\rho}_z + \widehat{w\rho}. \tag{13}$$

In addition,

$$\bar{p}_z + g\bar{\rho}/\rho_0 = 0, \tag{14}$$

$$\nabla\cdot\mathbf{U} + W_z = 0. \tag{15}$$

The velocities defined in (6) and (7) extend the usual definition of the residual-mean meridional circulation [see Andrews et al. (1987), p. 128] to three dimensions and to including zonal variations instead of

being defined as a zonal average. Equations (8)–(10) similarly extend the usual definition of the transformed Eulerian-mean set. Equations (11)–(13) extend the usual definition of Eliassen–Palm fluxes to both components of the momentum equation rather than just the zonal component and add x terms to these fluxes.

We now assume that mesoscale eddy density fluxes are aligned along the mean isopycnals and do not have a component perpendicular to them. This assumption has been made in our previous work, is discussed in full in Gent and McWilliams (1990), and means that G, defined in (13), is zero. Alternatively, McDougall and McIntosh (1996b, manuscript submitted to *J. Phys. Oceanogr.*) have recently shown how G can be made very small when the filtering operator is the familiar time average at constant z. They define

$$\widehat{\mathbf{u}\rho} = \overline{\mathbf{u}'\rho'} - \frac{1}{2}\,\bar{\mathbf{u}}_z\overline{\rho'^2}/\bar{\rho}_z, \tag{16}$$

where here the overbar and prime mean a time average and deviation from it. McDougall and McIntosh call the velocity, defined by substituting (16) into Eqs. (6) and (7), the temporal residual-mean velocity. They then show that

$$\frac{D^*}{Dt}\left[\bar{\rho} - \frac{1}{2}\,(\overline{\rho'^2}/\bar{\rho}_z)_z\right] = O(\alpha^3), \tag{17}$$

where α is the disturbance amplitude, which is assumed to be small. Further analysis and interpretation of Eqs. (16) and (17) is in McDougall and McIntosh (1996a,b, manuscripts submitted to *J. Phys. Oceanogr.*).

The GM parameterization gives a form for the tracer equation to be used in non-eddy-resolving ocean circulation models. When the flow is adiabatic and potential density is a simple function of potential temperature and salinity, this reduces to parameterizing the density equation. Gent et al. (1995) assume that eddy density fluxes act along, and not perpendicular to, isopycnal surfaces. This means that G in Eq. (13) is set to zero by assumption, in contrast to the generalization of this assumption due to McDougall and McIntosh described above. Thus, Gent et al. (1995) write the density equation in the form

$$\frac{D^*}{Dt}\,\bar{\rho} = 0, \tag{18}$$

where $(\mathbf{U}, W)$ is the mass-weighted filtered velocity, filtered at constant density rather than at constant depth. Gent et al. (1995) call $(\mathbf{U}, W)$ the effective transport velocity after Plumb and Mahlman (1987), but it can also be interpreted as the extended residual-mean meridional circulation defined in (6) and (7). Further discussion and interpretation of these three velocities is in Andrews et al. (1987) and McDougall and McIntosh (1996a,b, manuscripts submitted to *J. Phys. Oceanogr.*).

There have been previous generalizations of the residual-mean circulation and Eliassen–Palm fluxes to three dimensions. Hoskins (1983) and Plumb (1986) consider quasigeostrophic three-dimensional flows and define a horizontal residual-mean circulation that has more terms than that in Eq. (6). The additional terms are included so that the Eliassen–Palm flux divergences are precisely the eddy fluxes of the quasigeostrophic potential vorticity. This is based on the idea of potential vorticity mixing applied to the transformed Eulerian equations. However, quasigeostrophy is a special case in that potential vorticity is linear and so has simple quadratic flux terms, and advection in the substantial derivative is by the geostrophic velocity. It is very difficult, if not impossible, to generalize this idea to the 3D primitive equations, because potential vorticity is a nonlinear quantity and the substantial derivative has advection by the full velocity. We prefer our simpler definition of the residual-mean circulation because of the simple form of Eqs. (8)–(13). The implications of our assumptions on the full primitive equation potential vorticity are discussed in the next section. This 3D quasigeostrophic work is extended by Plumb (1990) and Andrews (1990).

Trenberth (1986) generalized the Hoskins and Plumb ideas to the 3D primitive equations. However, his definition of the residual-mean circulation is different than both the Hoskins and Plumb form and Eq. (6). He defines the transformed Eulerian-mean equations using the usual substantial derivative, D/Dt, rather than the modified form, D^*/Dt, as in (8)–(10). We prefer our simpler definition of the residual-mean circulation, and having advection in the transformed Eulerian-mean equations by $(\mathbf{U}, W)$, in part because of the resulting representation of nonacceleration relations, see section 4.

If Eqs. (8) and (9) are transformed to isopycnal coordinates, they are the same as the momentum equation (2.14) in the independent work of Lee and Leach (1996). These authors define their filtering operator to be a time mean, so their $(\mathbf{U}, W)$ is the mass-weighted time-mean velocity and time derivatives drop out. The equivalent of the extended Eliassen–Palm fluxes (11) and (12) is their Eq. (2.10), which also contributes to both components of the momentum equation and has added x components. Lee and Leach (1996) analyze the results from an eddy-resolving numerical model that uses isopycnal coordinates and has five layers. They simulate the eddies that form on a free jet over a period of 5 years and analyze the results from the second model layer. It is clear from their Figs. 11 and 12 that the pointwise Eliassen–Palm flux divergences on the right-hand side of their Eq. (2.14) are not small in the time-mean balance of the momentum equation. This will also be true in Eqs. (8) and (9), so now we address the question of how to parameterize these equations.

3. Parameterizing the momentum equation

The traditional parameterization of the eddy momentum terms in Eq. (5) in z-coordinate numerical

models is downgradient diffusion in the horizontal and vertical directions. In the horizontal either Laplacian or biharmonic operators are mostly used with a coefficient much larger than would be used in an eddy-resolving calculation, as required for computational stability on a coarser grid. The vertical viscosity used in non-eddy-resolving models is usually only a little larger, or the same, as in eddy-resolving calculations because it is believed to be primarily associated with turbulent motions on scales even smaller than the mesoscale. However, we think that the momentum equation parameterization should be guided by the Eliassen–Palm flux forms presented in the previous section. All three equations (8)–(10) use the modified substantial derivative D^*/Dt. The GM parameterization retains this advection in the tracer equation (18), so it is much more consistent to retain this advection in the momentum equation. Doing so presumes a principle of momentum conservation on mean Lagrangian trajectories, apart from the additional effects of Coriolis, pressure, and turbulent forces in (8) and (9).

Therefore, we propose to parameterize the E–P flux divergences in z-coordinate models as almost downgradient horizontal and vertical momentum diffusion plus the appropriate Coriolis term; an isopycnal coordinate alternative is described in the appendix. Thus, we propose in Eqs. (11) and (12)

$$\mathbf{E} = [\nu_H(\bar{u}_x - \bar{v}_y),\ \nu_H(\bar{u}_y + \bar{v}_x),\ \nu_V\bar{u}_z + f\widehat{v\rho}/\bar{\rho}_z], \tag{19}$$

$$\mathbf{F} = [\nu_H(\bar{v}_x + \bar{u}_y),\ \nu_H(\bar{v}_y - \bar{u}_x),\ \nu_V\bar{v}_z - f\widehat{u\rho}/\bar{\rho}_z]. \tag{20}$$

The resulting parameterized non-eddy-resolving momentum equation is

$$\frac{D^*}{Dt}\mathbf{u} + f\mathbf{k}\times\mathbf{u} + \nabla p = \nabla\cdot(\nu_H\nabla\mathbf{u}) + J_{xy}(\nu_H, \mathbf{k}\times\mathbf{u}) + (\nu_V\mathbf{u}_z)_z \equiv P(\mathbf{u}), \tag{21}$$

where J is the Jacobian operator and, for convenience, overbars have been dropped. Equation (21) has two changes from the traditional equation. The first change is that advection in the substantial derivative is by the residual-mean velocity. This is equivalent to parameterizing the eddy momentum terms in Eq. (5) as an additional advection by the eddy-induced velocity as well as almost downgradient diffusion. The second change is the Jacobian term on the right-hand side, which involves spatial gradients of ν_H. Wajsowicz (1993) has shown that, with the shallow-fluid approximation used with hydrostatic balance, this term is necessary to ensure that no stress is generated as a result of uniform rotation.

The kinetic energy density equation from (21) takes the form

$$\frac{D^*}{Dt}\left(\frac{\mathbf{u}\cdot\mathbf{u}}{2}\right) + \nabla^{3D}\cdot\mathbf{H} + g\rho w/\rho_0 = -\nu_H[(v_x + u_y)^2 + (u_x - v_y)^2] - \nu_V\mathbf{u}_z\cdot\mathbf{u}_z, \tag{22}$$

where $\mathbf{H}$ has components from the pressure gradient and diffusively parameterized terms. Equation (22) has divergence terms that integrate to zero globally, the usual transfer to potential energy term, and two terms on the right-hand side that are sinks of kinetic energy. These terms mimic the effect of barotropic instability in an eddy-resolving calculation, which transfers mean kinetic energy to eddy kinetic energy. It is shown in Gent et al. (1995) that, with the GM choice of eddy-induced velocity, the potential energy density equation has the same energy transfer term as in (22), plus a sink of potential energy. This last term mimics the effect of baroclinic instability in an eddy-resolving calculation, which transfers mean potential energy to eddy potential energy.

The equation for filtered potential vorticity can be formed from Eqs. (8), (9), and (18) and takes the form

$$\frac{D^*}{Dt}[\rho_z(f + v_x - u_y) - \rho_x v_z + \rho_y u_z] = \nabla^{3D}\cdot(\rho\mathbf{K}), \tag{23}$$

where the vector $\mathbf{K}$ is defined by

$$\mathbf{K} = \begin{pmatrix} -(\nabla^{3D}\cdot\mathbf{F})_z + J_{yz}(u, u^*) + J_{yz}(v, v^*) \\ (\nabla^{3D}\cdot\mathbf{E})_z - J_{xz}(u, u^*) - J_{xz}(v, v^*) \\ (\nabla^{3D}\cdot\mathbf{F})_x - (\nabla^{3D}\cdot\mathbf{E})_y + J_{xy}(u, u^*) + J_{xy}(v, v^*) \end{pmatrix}; \tag{24}$$

$\mathbf{E}$ and $\mathbf{F}$ are defined in (19) and (20), and $\mathbf{u}^* = \mathbf{U} - \mathbf{u}$ is the eddy-induced transport velocity. Equation (24) shows how the flux terms in the potential vorticity equation depend on the Eliassen–Palm fluxes in the momentum equation. It can be shown from Eq. (23) that potential vorticity is conserved both on isopycnal surfaces and in the total volume, except for possible boundary sources.

If the quasigeostrophic limit is taken in (23), by assuming that the Rossby number is small, the equation takes the form (for constant ν_H and ν_V)

$$\frac{D^g}{Dt}[f + \nabla^2 p/f_0 + f_0(p_z/N^2)_z] = (\nabla^{3D}\cdot\mathbf{F}^g)_x - (\nabla^{3D}\cdot\mathbf{E}^g)_y = P(\nabla^2 p)/f_0 + \nabla\cdot(f\mathbf{u}^*), \tag{25}$$

where the superscript g indicates a geostrophic approximation and N is the buoyancy frequency. Here D^g/Dt can include horizontal advection by a geostrophic component of $\mathbf{u}^*$, which is defined such that its horizontal divergence is zero. However, the horizontal divergence of $f\mathbf{u}^*$ on the right-hand side of Eq. (25) cannot be

omitted because it is leading order in Rossby number (see below). Equation (25) extends the usual relationship between quasigeostrophic potential vorticity and the Eliassen–Palm fluxes to the situation where there are x variations.

The GM choice of the eddy-induced velocity is

$$\mathbf{u}^* = (\kappa \nabla \rho / \rho_z)_z. \tag{26}$$

We can make a geostrophic scaling estimate of its magnitude as $\mathbf{u}^* \sim \kappa V / f R_d^2$, where V is a typical horizontal velocity, $R_d = NH/f$ is the baroclinic deformation radius, and H is a characteristic vertical scale. A common estimate for κ is $\sim V R_d$, based upon the baroclinic instability process, for example, Visbeck et al. (1996, manuscript submitted to *J. Phys. Oceanogr.*), in which case $\mathbf{u}^*/V \sim V/fR_d$, which is a Rossby number. This estimate justifies the ordering assumption behind Eq. (25). Note also that this choice for the eddy-induced velocity has comparably large horizontal divergence and vertical vorticity. Thus, it is appropriate to interpret it as contributing to the dynamics more as an ageostrophic velocity than a geostrophic velocity. It is of primary importance in the zonally averaged dynamics, where ageostrophic advection is dominant, but its largest contribution to 3D pointwise dynamics is as an additional vortex stretching effect, and not as a significant advection velocity, as in Eq. (25).

By setting κ constant and using the quasigeostrophic approximation in (26), the quasigeostrophic potential vorticity equation (25) becomes

$$\frac{D^g}{Dt}[f + \nabla^2 p/f_0 + f_0(p_z/N^2)_z] = \nabla \cdot [\nu_H \nabla(\nabla^2 p)/f_0 + ([\kappa f/N^2 + \nu_V/f_0]\nabla p_z)_z]. \tag{27}$$

Therefore, if $\nu_H = \kappa$ and if ν_V and diffusion of the Coriolis term are neglected, Eq. (27) shows that quasigeostrophic potential vorticity is advected by the geostrophic velocity and diffused by a Laplacian operator. An assumption of this form is the basis for the potential vorticity homogenization theory of Rhines and Young (1982). This is also the basis for the concept of potential vorticity mixing that led to the Hoskins (1983) and Plumb (1986) choice for the residual-mean meridional circulation. Their choice automatically gives a quasigeostrophic potential vorticity equation of the Rhines and Young form. Our momentum equation parameterization does not assure that quasigeostrophic potential vorticity is diffused exactly. However, we have shown here that a rather similar equation results from our simpler choice of the residual-mean meridional circulation and the GM parameterization.

4. The Antarctic Circumpolar Current and the Deacon cell

Andrews et al. (1987, pp. 131 and 132) shows that atmospheric application of Eliassen–Palm fluxes based on zonal averaging led to generalized Eliassen–Palm and nonacceleration relations. When these relations are satisfied, eddy effects on the mean circulation can be completely described by the transformed Eulerian-mean equations, which have a steady solution with zero residual-mean circulation and zero zonally integrated Eliassen–Palm flux divergence. Necessary conditions for these relations to hold are that the dynamics are adiabatic and inviscid and the pressure gradient term is zero, which occurs in the zonal momentum equation for zonal averaging and a periodic domain.

The only region of the World Ocean that has strong currents and is zonally periodic is the Antarctic Circumpolar Current (ACC). Therefore, we can anticipate that nonacceleration relations will apply to the ACC, at least to some degree. The ACC has large zonally standing meanders, and northward and southward excursions at the same density can occur at considerably different depths. A consequence of this is that the standard meridional overturning streamfunction shows quite a strong overturning cell near the ACC. The streamfunction is defined by

$$\overline{\psi^z}(y, z) = \int_z^0 \overline{v}(x, y, z')dz', \tag{28}$$

where the overbar here indicates a zonal and time average, and the cell is usually called the Deacon cell. If the interior motions below the surface boundary layer are assumed to be almost adiabatic on long timescales, then a zonal and time average of the thickness equation in isopycnal coordinates, which expresses mass balance in isopycnal layers, gives

$$(\overline{h_\rho v})_Y \approx 0. \tag{29}$$

Here the overbar is a zonal and time average at constant ρ, h_ρ is the thickness of a density layer, and subscripts X and Y indicate differentiation at constant density rather than at constant z. Equation (29) expresses the absence of any net meridional mass flux within an isopycnal layer. A consequence of this is that if the meridional overturning streamfunction in (28) is calculated down to a constant density surface rather than to a constant depth, then the strong Deacon cell disappears. This has been shown recently by Döös and Webb (1994) who analyzed results from the Fine Resolution Antarctic Model (FRAM). For subsurface layers that have no significant diabatic mass fluxes, (29) can be integrated to obtain

$$\bar{V} \equiv \bar{v} + \overline{h'_\rho v'}/\bar{h}_\rho \approx 0, \tag{30}$$

where a decomposition into a mean meridional velocity and an eddy mass flux term has been made, so that the prime here denotes departures from the zonal and time average. If the zonal momentum equation in isopycnal coordinates is averaged in the same way, then it takes the form

$$-f\bar{h}_\rho \bar{V} = \mathcal{E}_Y^{(y)} + \mathcal{E}_\rho^{(\rho)} + \mathcal{D}^{(x)}. \tag{31}$$

Here $\mathcal{D}^{(x)}$ is the mean zonal viscous term and

$$\mathcal{E}^{(y)} = -\overline{h_\rho u v}, \quad \mathcal{E}^{(\rho)} = \overline{M h_X}, \tag{32}$$

where M is the Montgomery potential. The nonlinear flux terms in (32) are the Eliassen–Palm fluxes appropriate to this averaging operator; they are the lateral Reynolds stress and the diapycnal form stress acting on isopycnal surfaces, respectively. Note the mathematical similarity between Eqs. (31) and (32) and the zonally integrated forms of Eqs. (8) and (11). For nearly adiabatic motions, (30) implies that the left-hand side of (31) is approximately zero. So, for small $\mathcal{D}^{(x)}$, the Eliassen–Palm flux divergence must also be approximately zero. Thus, the nonacceleration conditions will apply to the interior parts of the ACC with almost conservative dynamics.

Products of the mean fields do not contribute significantly to the nonlinear fluxes in (32) because $\bar{v}$ is ageostrophic and $\bar{h}_X$ is zero, so that the $\mathcal{E}$ are primarily due to eddy fluxes. It has long been believed that a significant part of the eddy fluxes in the ACC come from the large-scale standing meanders, as well as from smaller-scale standing and transient eddies; see Gill and Bryan (1971), McWilliams et al. (1978), McWilliams and Chow (1981), Treguier and McWilliams (1990), Wolff et al. (1991), Döös and Webb (1994), Killworth and Nanneh (1994), and McIntosh and McDougall (1996). Here $\mathcal{E}$ contains both classes of eddies. In any 3D ocean model calculation, some fraction of the eddy transports will be on large enough scales to be part of the calculated solution, and the remaining small-scale contributions must be parameterized. The partition between these two classes will shift with the model grid resolution, especially between models that resolve mesoscale eddies and those that do not.

We now discuss these balances in non-eddy-resolving solutions using the GM parameterization. The equivalent to (29) is just $\bar{V}_Y \approx 0$, where V is now the thickness-weighted velocity defined in (30) (see Gent et al. 1995). Therefore, the zonally and time-averaged meridional overturning streamfunction calculated down to a density surface,

$$\overline{\psi^\rho}(y, \rho) = \int_\rho^0 \bar{V}(x, y, z')dz', \tag{33}$$

will be very close to zero in conservative regions of the ACC. Hirst and McDougall (1996b, manuscript submitted to *J. Phys. Oceanogr.*) show that this streamfunction is indeed almost zero in the interior in non-eddy-resolving solutions using GM. This is consistent with the Döös and Webb (1994) result that the Deacon cell disappears when the streamfunction integration is down to a constant density surface. The Deacon cell is present if the streamfunction in (33) is calculated using the zonally and time-averaged Eulerian-mean velocity, $\bar{v}$, instead of $\bar{V}$. It remains to discuss the more usual meridional overturning streamfunction in non-eddy-resolving models where the integration is down to a constant depth, rather than to a constant ρ. This streamfunction will include the effects of the parameterized small-scale standing and transient eddies, but not those of the resolved standing meanders of the ACC. Thus, only partial cancellation of the Deacon cell is to be expected. However, in the non-eddy-resolving solution using GM of Danabasoglu et al. (1994), it was found that the Deacon cell virtually disappeared in the interior, even though only the parameterized eddy contribution is included. Subsequent solutions of Danabasoglu and McWilliams (1995) and Hirst and McDougall (1996a,b, manuscript submitted to *J. Phys. Oceanogr.*) have shown that the cancellation is not exact and the Deacon cell is only partially cancelled. The precise degree of cancellation varies with the parameters of the calculation, the model topography, and other factors, although it surprisingly seems that the majority of the Deacon cell is cancelled in the ocean interior.

Many assumptions are invoked in the preceding arguments, including conservative dynamics, similar properties of different averaging operators, and positive reinforcement between resolved and parameterized eddies in the ACC. More refined assessments of the degree of validity of these assumptions are needed. Nevertheless, these arguments lead to the conclusion that approximate nonacceleration relations are occurring in both eddy-resolving and non-eddy-resolving model solutions in the zonally periodic, interior regions of the ACC. These relations include a weak residual-mean meridional circulation, with partial cancellation of the Deacon cell, and nearly nondivergent Eliassen–Palm fluxes. The GM tracer parameterization and the momentum equation parameterization proposed in section 3 provide integrally consistent representations for the role of unresolved eddies in these relations.

5. Conclusions

The relevant averaging operator when considering ocean eddies is a low-pass filter in time and space rather than time or zonal averaging. We have shown that the concepts of residual-mean circulation, transformed Eulerian-mean equations, and Eliassen–Palm fluxes can be generalized to include x and t variations and both components of the momentum equation. This complements the recent work of Lee and Leach (1996) who considered the effects of time averaging on the momentum equation in isopycnal coordinates. They also show Eliassen–Palm fluxes with x variations in both components of the momentum equation.

We then make a proposal to parameterize the Eliassen–Palm fluxes in z-coordinate models as almost downgradient diffusion of the filtered velocity plus the appropriate Coriolis term. This results in a momentum equation (21) for non-eddy-resolving ocean models that differs in two ways from the traditional momentum equation. The first is that advection is by the residual-

mean velocity rather than by the filtered velocity. The second is a term proportional to horizontal gradients of ν_H that Wajsowicz (1993) showed is required so that uniform rotation does not produce a stress. Use of momentum equation (21) and tracer equation (18) is consistent because advection in both equations is by the residual-mean velocity. Using the GM choice for the eddy-induced transport velocity, the equation set gives a total energy budget that has sinks of kinetic and potential energy that mimic the effects of barotropic and baroclinic instability, respectively, in an eddy-resolving model.

The potential vorticity budget for the non-eddy-resolving model is formed and the dependence on the Eliassen–Palm fluxes is made explicit. The quasigeostrophic limit is taken and the equation presented with the GM choice of eddy-induced transport velocity. This equation is close to showing that quasigeostrophic potential vorticity is advected by the geostrophic velocity and diffused by a Laplacian operator. An assumption of this form is the basis for the potential vorticity homogenization theory of Rhines and Young (1982). The quasigeostrophic potential vorticity equation in isopycnal coordinates is given in the Appendix.

Döös and Webb (1994) analyzed the eddy-resolving FRAM calculation. They show that the Deacon cell, which appears in the usual meridional overturning streamfunction, disappears when v is integrated down to a constant density surface rather than a constant depth. The non-eddy-resolving equivalent of this result in models with GM is that the mean meridional overturning streamfunction, calculated by integrating V down to a constant density disappears (see Hirst and McDougall 1996b, manuscript submitted to *J. Phys. Oceanogr.*). The overturning streamfunction calculated from the Eulerian-mean velocity, v, shows the usual Deacon cell. However, it is partially cancelled if V is integrated down to a constant depth; see Danabasoglu et al. (1994) and Danabasoglu and McWilliams (1995). The partial cancellation of the Deacon cell in non-eddy-resolving ocean models is equivalent to the nonacceleration relations for zonally averaged atmospheric flow, described in Andrews et al. (1987) for example. This extends the recent work of Tandon and Garrett (1996), who showed the same result in purely 2D flow, which is simpler because there are no standing eddies.

We expect that the proposed momentum equation (21) will make little difference to coarsely resolved global ocean model solutions with uniform ν_H, such as those of Danabasoglu et al. (1994), Danabasoglu and McWilliams (1995), and Hirst and McDougall (1996a,b, manuscript submitted to *J. Phys. Oceanogr.*). The reason is that the only difference would be in the inertial term, which is mostly small when the major boundary currents have a primarily diffusive balance. Of course, the eddy-induced momentum advection term can become larger when the ocean model has finer resolution and the major currents have inertially dominated dynamics. Nevertheless, we think it is much more consistent that non-eddy-resolving ocean models in z coordinates use the momentum equation (21), rather than the usual form with advection by the Eulerian mean velocity.

Finally, we think that the nearly downgradient parameterization forms in (19) and (20), and in the GM tracer parameterization, are undoubtedly too simple to represent eddy effects realistically. Alternatives should be considered in the near future, probably based upon eddy-resolving model solutions. However, we believe that momentum and tracer eddy flux divergences from these solutions must be analyzed in a way that allows for an additional eddy-induced advection term.

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APPENDIX

Isopycnal Coordinates

If the potential vorticity equation (23) is transformed to isopycnal coordinates, then it takes the simpler form

$$\frac{D^*}{Dt}\left[\frac{f + v_X - u_Y}{h_\rho}\right] = [(\nabla^{3D}\cdot\mathbf{F})_X - (\nabla^{3D}\cdot\mathbf{E})_Y + J_{XY}(u, u^*) + J_{XY}(v, v^*)]/h_\rho, \quad \text{(A1)}$$

where h_ρ is the filtered thickness and subscripts X, Y indicate differentiation at constant density rather than constant z. Equation (A1) generalizes Eq. (3.10) of Lee and Leach (1996). They show that when the flow is quasigeostrophic, then the divergence terms of the extended Eliassen–Palm fluxes act like eddy terms for a linear approximation to the exact potential vorticity. Their equation is a generalization of the usual quasigeostrophic result.

In ocean models that are formulated in isopycnal coordinates, the momentum diffusion used has been oriented along and normal to isopycnal surfaces using the small slope approximation. This is much simpler to implement, and it is assumed that the orientation of the dissipation does not greatly affect the solutions. This dissipation form should be extended to include the terms suggested by Wajsowicz (1993), as in Eqs. (19) and (20), so that uniform rotation does not generate a stress. If the quasigeostrophic limit is taken and the GM eddy-induced transport velocity is used, then the potential vorticity equation (A1) becomes (for ν_H and κ constant and neglecting the ν_V term)

$$\frac{D^g}{Dt}[f + \nabla^2 M/f_0 + f_0 S M_{pp}]$$

$$= \nabla\cdot[\nu_H \nabla(\nabla^2 M)/f_0 + \kappa f S \nabla M_{pp}], \quad \text{(A2)}$$

where the Montgomery potential M and S are given by

$$M = p + g\rho h/\rho_0, \quad S = (\rho_0 N/g)^2. \quad \text{(A3)}$$

Equation (A2) is the isopycnal coordinate form of equation (27), with ν_V set to zero. It again shows that, if $\nu_H = \kappa$ and diffusion of the Coriolis term is neglected, quasigeostrophic potential vorticity is advected by the geostrophic velocity and diffused by a Laplacian operator.

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